

Exploring Factors: Odd, Even, Prime, Composite and Squares

Overview

This lesson will focus on finding factors of numbers under 100 and making conjectures about them.

Benchmarks 3 – 4

Number, Number Sense and Operations

G. Recognize and classify numbers as prime or composite and list factors.

Grade 4 Indicators

Identify and represent factors and multiples of whole numbers through 100, and classify numbers as prime or composite

Mathematical Processes

- E. Link concepts to procedures and symbolic notation; e.g., model 3×4 with a geometric array, represent one-third by dividing an object into three equal parts.
- I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem are more helpful than others.
- J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.

Benchmarks 5 - 7

Number, Number Sense and Operations

G. Apply and explain the use of prime factorizations, common factors, and common multiples in problem situations.

Grade 5 Indicators

Recognize and identify perfect squares and their roots.

This indicator is also aligned to Grade 8-10 Number, Number Sense and Operations Benchmark H.

Grade 6 Indicators

Decompose and recompose whole numbers using factors and exponents (e.g., $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$), and explain why "squared" means "second power" and "cubed" means "third power."

Find and use the prime factorization of composite numbers. For example:
Use the prime factorization to recognize the greatest common factor (GCF).
Use the prime factorization to recognize the least common multiple (LCM).
Apply the prime factorization to solve problems and explain solutions.

Grade 7 Indicators

Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents and square roots (for perfect squares).

Mathematical Processes

H. Use representation to organize and communicate mathematical thinking and problem solutions.

Benchmarks 8 – 10

H. Find the square root of perfect squares, and approximate the square root of non-perfect squares.

Grade 8 Indicators

Find the square root of perfect squares, and approximate the square root of non-perfect squares as consecutive integers between which the root lies;

Materials

Students will need a set of about 64 square tiles (or Cheese-It crackers) for each pair or group of three students, worksheets, legal size paper, scotch tape and markers.

Procedure:

- Ask students, in pairs or groups of three, to create rectangular arrays and list all pairs of numbers that multiply to make 12. Have the students share their responses with a partner or group. Informally assess by walking around the room listening for students to identify multiple arrays for 12. Have students do the same task with the number 24.
- Ask student pairs, or groups of three, to describe what they know about: factors, products, odd, even, prime and composite numbers. Stress that it is okay if the students cannot define these words or have never heard them. Have volunteers share their responses with the class, and have the class reach a consensus on the definitions of factors and products, odds, evens, prime and composite number and record definitions on board.

- Ask students, in pairs or groups of three, to create rectangular arrays and list all pairs of numbers that multiply to make 2-11 and 64. Have students demonstrate, using smart notebook or an overhead, making as many arrays as possible out of the numbers 1 – 11 and 64. Discuss how going in a logical order may help to not miss any arrays. Have students create a poster for each number. Give students time to fill out their charts as they go along. The teacher will demonstrate how she wants each factor poster set up. Be sure to have the students pay attention to whether one of rectangles created is a square and mark the poster with it.
- Students will then continue in pairs or groups of 3, with making arrays and finding factors for 13 through 25. Informally assess by walking around the room listening to students and looking at their charts.
- Each group will be assigned about 3-5 numbers to cover 13 through 63. A poster will be made of each number showing its factors, odd or even, composite or prime and if a square was created.
- Posters will be taped in order around the room. Posters will be examined and conjectures will be made. Then have students reorganize by odd and even. Students then will suggest other ways we can order posters. Keep re-ordering them and making conjectures. Allow students time between each ordering to discuss and record their conjectures.

Commentary:

Involving students in making conjectures helps make important mathematical ideas explicit for students and provides opportunities for students to communicate using precise mathematical language (Carpenter, Franke, & Levi, 2003). Making mathematical ideas explicit is important, especially for ideas that seem obvious to adults. For example, when starting a lesson by stating the definition of prime numbers, a teacher may think that the connection to the number of factors is obvious. Students, however, may not make that connection. Allowing students to make conjectures after observing the behavior of prime numbers in creating factor pairs makes explicit for students that connection. Also, providing students opportunities to make conjectures creates a rich environment in which students practice the use of mathematical language.

Instructional Tip:

Students may wonder why the number zero has not been discussed in the prime and composite discussion. Including in the definition of primes and composites that these refer only to natural (counting) numbers may be helpful. Prime and composite vocabulary is not applied to the whole numbers. The whole numbers include zero, whereas the natural numbers do not include zero.

Have students discuss whether they think the number one is prime or composite.

- Have students write all the different factors of one.
- Have students read the formal definition of prime and composite from earlier in the lesson. Ask the students if the number one fits either of these definitions?"(No) "Why or why not?" (One has only one unique factor, not exactly two, and not more than two.)
- Define 1 (one) as a special case – 1 (one) is a number that is neither prime nor composite. It is the only number with this property.

Duration:

2 Blocks (80 minutes each)

Extensions:

The Sieve of Eratosthenes

Have students act out the method the ancient Greeks used to determine if a number was a prime or not: The Sieve of Eratosthenes. Use Attachment D, *Sieve of Eratosthenes*, to have students act out the method using the following steps:

- a. Have students count off consecutively, or hand students a numbered card. The first student is 2; second student is 3, et cetera.
- b. All students are standing.
- c. Each student who have a number larger than 2 that is divisible by 2 sits down and remains seated for the remainder of the game.
- d. Each student whose number is larger than three and is divisible by three sits down and remains seated for the remainder of the game.
- e. Each student whose number is larger than four and is divisible by four sits down and remains seated for the remainder of the game.
- f. Each student whose number is larger than five and is divisible by five sits down and remains seated for the remainder of the game.
- g. Each student whose number is larger than six and is divisible by six sits down and remains seated for the remainder of the game.
- h. The class should be reduced to only prime numbers standing. Have students write down the numbers of those students standing. Have students check that these numbers are indeed prime by listing the factors.
- i. Give the students the *Sieve of Eratosthenes*, Attachment E and go through the process again, marking off numbers instead of students being seated.
- j. Have groups or pairs compare lists and check for accuracy. The list should be: 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,97,101
- l. Remind students of their earlier discovery that two is the only even prime number. Discuss how the Sieve of Eratosthenes informally confirms this conclusion. Is one prime or composite?"

The Factor Game, Connected Mathematics Prime Time, Pearson Prentice Hall

The Factor Game engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and

numbers with few factors. Students are then guided through an analysis of game strategies and introduced to the definitions of prime and composite numbers.

The Product Game, Connected Mathematics Prime Time, Pearson Prentice Hall

Students learn how to play the Product Game. As they play the game, students develop understanding of factors, multiples, and the relationships between them. Winning strategies are discussed. The Product Game was adapted from *Prime Time: Factors and Multiples*, part of the Connected Mathematics Project, and was written by G. Lappan, J. Fey, W Fitzgerald, S. Friel and E. Phillips (Dale Seymour Publications, 1996, pp.17-25.)

The Product Game Unit by NCTM's Illuminations

In the Factor Game, students start with a number and find its factors. In the Product Game, students start with factors and multiply to find the product. The two games work well together because they help students to see the relationship between products and factors. There are four lessons in The Product Game unit by NCTM's Illuminations. Students play the product game in the first lesson. In the second lesson, they make their own game boards. The task of creating a new game is challenging to most students. They learn a lot by experimenting and by making mistakes about what factors and products to include in a game. In the third lesson, students use Venn diagrams to represent the relationships between the factors or products of two numbers. The fourth lesson contains extensions and connections.

Real World Application:

Locate items in the grocery that are packaged in a way representing composite numbers, and if possible, prime numbers. Ask students to explain why grocery items, such as eggs, are packed in sets of 12, not 11 or 13. Encourage them to consider the arrangement of the possible arrays.

Possible Assessments:

- Use *Describing What I Know*, Attachment A, for students to describe how to determine if a number is prime, composite or neither.
- Distribute *Prime and Composite Post-Assessment*, Attachment B, after students complete *Describing What I Know*, Attachment A. Allow students to use their descriptions from *Describing What I Know*, Attachment A as they complete *Prime and Composite Post-Assessment*, Attachment B.

Scoring Guidelines:

- Descriptions written for *Describing What I Know*, Attachment A, should include an explanation for the special case of one, a working definition of prime as having exactly two different factors, and a working definition of composite as having a list of more than two different factors.
- To score *Prime and Composite Post-Assessment*, Attachment B, use the *Prime and Composite Post-Assessment Rubric*, Attachment C.

Websites

Illuminations' The Factor Game

<http://illuminations.nctm.org/LessonDetail.aspx?ID=L620>

Illuminations' The Product Game

<http://illuminations.nctm.org/LessonDetail.aspx?ID=U100>

Connected Math Interactive Resources

<http://connectedmath.msu.edu/CD/Grade6/index.html>

The Math We Need to Know and Do in Grades 6-9 By Pearl Gold Solomon

http://books.google.com/books?id=uQh2YMgToulC&pg=PA115&lpg=PA115&dq=factors+composite+prime&source=web&ots=2CWQld0Qta&sig=mdhGZ0idczGwNID-E58JRe3N34s&hl=en&sa=X&oi=book_result&resnum=2&ct=result#PPA110,M1

Quia Games Dealing with Factors

<http://www.quia.com/shared/search>

PBS Mathline Factor Game

<http://www.pbs.org/teachers/mathline/lessonplans/pdf/msmp/factor.pdf>

BBC Grid Game

<http://www.bbc.co.uk/education/mathsfle/shockwave/games/gridgame.html>

Multiplication Mystery

<http://www.harcourtschool.com/activity/mult/mult.html>

wicked Multiple Matrix

http://www.tki.org.nz/r/wick_ed/maths/interactives_matrix.php

Smart-created Factors and Multiples

<http://www.education.smarttech.com/ste/en-US/Ed+Resource/Lesson+activities/Notebook+Activities/Browse+Notebook/United+States/Elementary/4-6/Math/Factors+and+Multiples.htm>

Smart-created Factors, Multiples and Primes

<http://education.smarttech.com/ste/en-US/Ed+Resource/Lesson+activities/Notebook+Activities/Browse+Notebook/United+States/Elementary/4-6/Math/Factors+Multiples+and+Prime+Numbers.htm>

Exploring Numbers

#	Factors	odd or even	Prime or Composite	Perfect Square? What is square root?
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				
13.				
14.				
15.				

16.				
17.				
18.				
19.				
20.				
21.				
22.				
23.				
24.				
25.				
26.				
27.				
28.				
29.				
30.				

Factor Game

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

Factor Game

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
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Factor Game

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Factor Game

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Factor Game

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Factor Game

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

Teach the Factor Game to a willing adult and play at least 3 rounds with them.

Factor Game

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

Factor Game

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

Willing Adult,
Good Luck!
Please sign at the bottom.

Factor Game

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
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Factor Game

1	2	3	4	5	6
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Factor Game

1	2	3	4	5	6
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13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

Attachment A

Describing What I Know

Name _____

Date _____

Directions: Inside each box, write at least 3 examples of each type of number. In the lines under the box, write how you determine if a number is that type.

PRIMES

Composites

Numbers that are neither
prime nor composite

Attachment B
Prime and Composite Post-Assessment

Directions: Determine if the following numbers are prime, composite, or neither. Explain how you arrived at your answer.

A. 1

B. 2

C. 9

D. 17

E. 21

F. 37

G. 93

Attachment C
Prime and Composite Post-Assessment Rubric

Name _____

Date _____

Identified all factors.	Categorized correctly based on factors listed.	Provided a reasonable explanation for selected category.	Comments

**Attachment D
Sieve of Eratosthenes**

Name _____

Date _____

	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102